Two New von Kries Based Chromatic Adaptation Transforms Found by Numerical Optimization

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Abstract: In this article, two new von Kries based chromatic adaptation transforms are proposed. The numerical optimization procedure adopted for deriving them simultaneously exploits the whole sets of data available and the existing chromatic adaptation transforms. Experimental results report several statistics to prove the effectiveness of our proposals with respect to the state-of-the-art. © 2010 Wiley Periodicals, Inc. Col Res Appl, 35, 184 – 192, 2010; Published online 14 January 2010 in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.20573

INTRODUCTION

Chromatic adaptation transforms (CATs) are able to predict corresponding colors. A pair of corresponding colors consists of a color observed under one illuminant, and another color that has the same appearance when observed under a different illuminant.[1] This research topic has been extensively studied given its importance for many industrial applications, such as the prediction of color inconstancy, the evaluation of the color rendering property of light sources, and the achievement of successful color reproduction under different light sources.[1,2]

A survey of several CATs are given by Fairchild in his book.[2] Luo and Hunt[3] proposed a modified Bradford transform,[4] which is included in CIECAM97s. Finlayson and Süsstrunk[5] have derived a transform based on sharpened sensors. Li *et al.*[1] derived a transform, known as CMCCAT2000, by fitting all the available corresponding color data sets, instead of just the Lam and Rigg set. Moroney *et al.*[6] proposed a modified CMCCAT2000 to be used with the CIECAM02 model. In 2004, the CIE TC 1–52 "Chromatic Adaptation Transforms"[7] tested thirteen chromatic adaptation transforms indicating four possible candidates for future CIE recommendations giving quite similar performances. The members of the CIE TC 1–52 were unable to agree to a single CAT as some of them required that the adopted transform must be theoretically based. Other members still agreeing that such objective is desirable, considered that was important to indicate a single CAT that should work as well as possible, even if only applicable to a limited range of conditions.

In this article, we propose two von Kries-based chromatic adaptation transforms that outperform or are statistically equivalent to the existing ones on all the corresponding color datasets available. These transforms are found by numerical optimization based on Particle Swarm Optimization. The key idea in our procedure is the simultaneous use of all the corresponding color data sets available and the predictions of the corresponding colors done using already defined CATs. Because several works treats these data as arising solely from visual adaptation, we will do so here too. In the long run, however, adaptation models should be derived and/or tested by data sets based on experiments that keep the test-patch tristimulus values constant when the light is changed in the wider visual field. Only under such conditions can visual adaptation effects be separately inferred.

For the first CAT proposed, to boost as much as possible the performances, objective function uses both Wilcoxon signed-rank tests and the perceptual error metrics ΔE^*_{ab} and ΔE^*_{94}. As shown in the experimental results section, the proposed CAT outperforms existing solutions. For the second CAT we add to the aforementioned terms in the objective function, a positivity constraint on its spectral responses in order to have stable

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color ratios across illuminants.[8] The fitting results are, in this case, only statistically equivalent to the best available CATs.

CHROMATIC ADAPTATION TRANSFORMS

Several chromatic adaptation transforms exist in the literature, most based on the von Kries model.[3] CIE XYZ tristimulus values $[X'\ Y'\ Z']^T$ are linearly transformed by a 3 $\times$ 3 matrix M_{CAT} to derive the post-adaptation cone responses under the first illuminant. The resulting values are independently scaled to get the postadaptation cone responses under the second illuminant. This transform is usually a diagonal matrix based on the postadaptation cone responses of the illuminants' white-point. To obtain CIE XYZ tristimulus values under the second illuminant $[X''\ Y''\ Z'']^T$, the postadaptation cone responses under the second illuminant are then multiplied by the inverse of matrix M_{CAT}.[9] This model is outlined in Eq. (1):

$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = [M_{\mathrm{CAT}}]^{-1} * \begin{bmatrix} R''_{\mathrm{w}}/R'_{\mathrm{w}} & & \\ & G''_{\mathrm{w}}/G'_{\mathrm{w}} & \\ & & B''_{\mathrm{w}}/B'_{\mathrm{w}} \end{bmatrix} * [M_{\mathrm{CAT}}] * \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad (1)$$

where $[R'_{\mathrm{w}}\ G'_{\mathrm{w}}\ B'_{\mathrm{w}}]$ and $[R''_{\mathrm{w}}\ G''_{\mathrm{w}}\ B''_{\mathrm{w}}]$ are computed from the XYZ tristimulus values of the first and second illuminants by multiplying their XYZ tristimulus values $[X'_{\mathrm{w}}\ Y'_{\mathrm{w}}\ Z'_{\mathrm{w}}]^T$ and $[X''_{\mathrm{w}}\ Y''_{\mathrm{w}}\ Z''_{\mathrm{w}}]^T$ by M_{CAT}.

All the comparisons made in this work are based on the von Kries chromatic adaptation model as outlined in Eq. (1), where full adaptation by the human observer is assumed. The chromatic adaptation transforms used in this work are reported in Table I, while the corresponding normalized spectral responses are plotted in Fig. 1.

EXPERIMENTAL DATA SETS

Luo and Hunt accumulated several data sets based on reflective stimuli and data sets based on monitor and projected stimuli, that were widely used to derive and to test the performance of various chromatic adaptation transforms[10] and color appearance models.[11] These data have been collected from the Colour Science Association of Japan (CSAJ),[12] Kuo and Luo,[13] Lam and Rigg,[4] Helson *et al.*,[14] LUTCHI,[15] Breneman,[16] and Braun and Fairchild,[17] for a total of 26, subsets which total 671 pairs of corresponding colors. The main features of these data sets are summarized in[1] and reported in Table II. These data sets are the same used by Süsstrunk *et al.*[9]

TABLE I. Short names and entries of the chromatic adaptation transforms used in this work.

CAT name		CAT entries
1	von Kries	$M_{\mathrm{vonKries}} = \begin{bmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0 & 0 & 1 \end{bmatrix}$
2	Bradford	$M_{\mathrm{BFD}} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0686 & 1.0296 \end{bmatrix}$
3	Sharp	$M_{\mathrm{Sharp}} = \begin{bmatrix} 1.2694 & 0.0988 & -0.1706 \\ -0.8364 & 1.8006 & 0.0357 \\ 0.0297 & -0.0315 & 1.0018 \end{bmatrix}$
4	CMCCAT2000	$M_{\mathrm{CMCCAT}} = \begin{bmatrix} 0.7982 & 0.3389 & -0.1371 \\ -0.5918 & 1.5512 & 0.0406 \\ 0.0008 & 0.239 & 0.9753 \end{bmatrix}$
5	CAT02	$M_{\mathrm{CAT02}} = \begin{bmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.0030 & 0.0136 & 0.9834 \end{bmatrix}$

DERIVATION OF THE NEW CHROMATIC ADAPTATION TRANSFORMS

In this section, we describe the method adopted to derive the new chromatic adaptation transform.

Let M_i, $i = 1\ldots5$ be the five CATs listed in Table I.

Let CCDS_j, $j = 1\ldots16$ be the 16 corresponding color data sets listed in Table II.

Let $\mathrm{WSRT}_{\Delta E}(\cdot)$ and $\mathrm{WSRT}_{\Delta E_{94}}(\cdot)$ be the Wilcoxon signed-rank test[18] scores, representing the number of times a transform performed best or was statistically the same (at the 95 percent confidence, according to the Wilcoxon signed-rank test) as the best transform using respectively the perceptual error metrics ΔE^*_{ab} and ΔE^*_{94}. The Wilcoxon signed-rank test can be used to test the null hypothesis that two CATs have the same performance expressed as the median values μ_X and μ_Y of their error distributions X and Y, i.e., $H_0 : \mu_X = \mu_Y$. To test H_0, we consider the difference of independent error pairs $(X_1 - Y_1), \ldots, (X_N - Y_N)$ for N different corresponding color pairs. We rank these error pairs according to their absolute differences. If H_0 is correct, the sum of the ranks W will approximate zero. If W is much larger or smaller than zero, the alternative hypothesis $H_1 : \mu_X > \mu_Y$ or $\mu_X < \mu_Y$ is true. We can test H_0 against H_1 at a given significance level α. We reject and accept if the probability of observing the error differences we obtained is less than or equal to α. As already said before, in this work a significance level $\alpha = 0.05$ has been chosen. Comparing

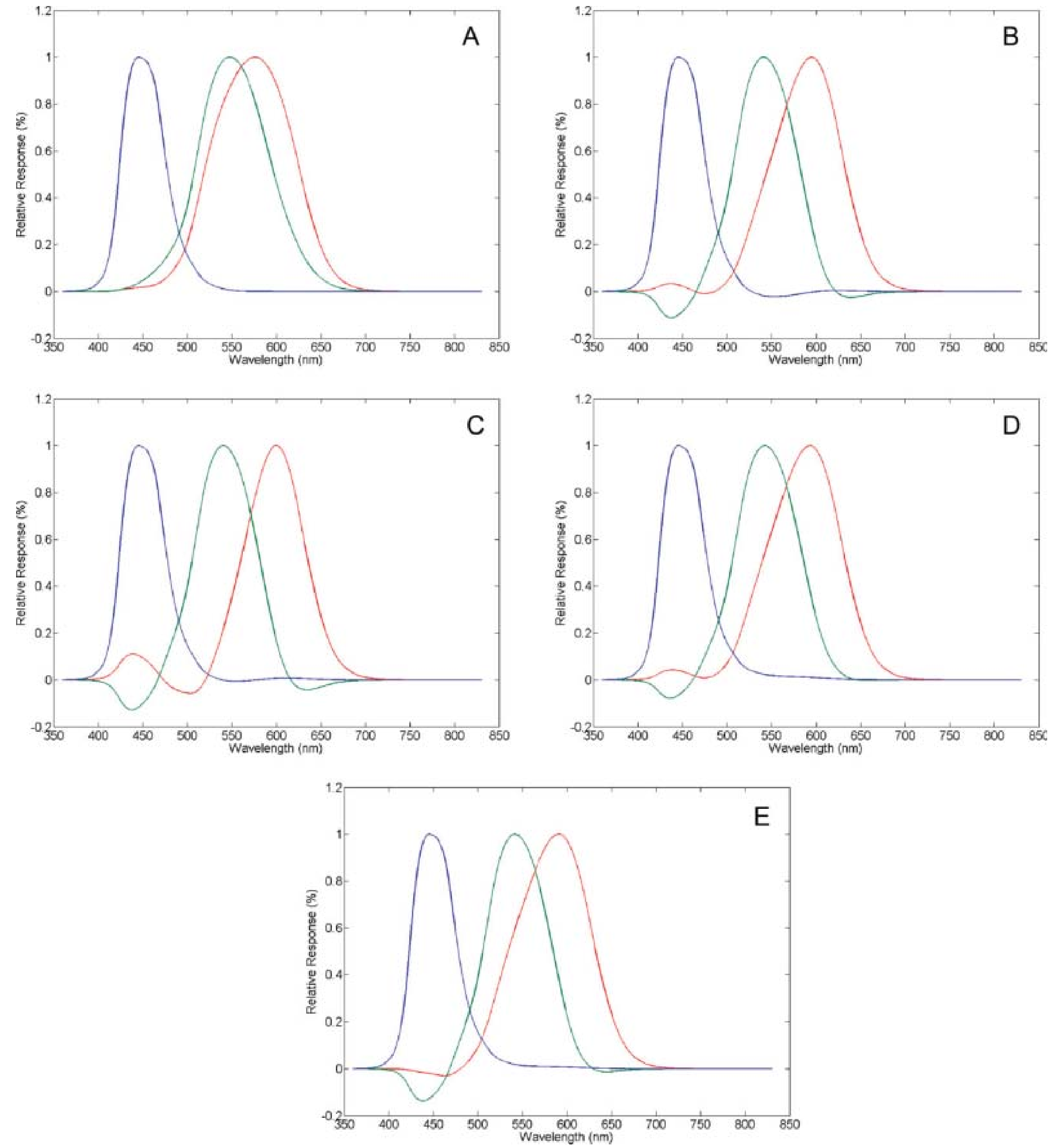

FIG. 1. Normalized spectral responses of the chromatic adaptation transforms implemented in this work. (A) von Kries; (B) Bradford; (C) Sharp; (D) CMCCAT2000; (E) CAT02. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

every CAT with all the others, we generated a score representative of the number of times that the null hypothesis H_0 has been rejected for the given CAT, i.e., the number of times that the performance of the given CAT has been considered to be better than the others.

Let $\Delta\mathrm{WSRT}_{\Delta E}(M)$ and $\Delta\mathrm{WSRT}_{\Delta E_{94}}(M)$ be the difference between the Wilcoxon signed-rank score of a generic transform M and the maximum score obtained by the five CATs M_i, $i = 1 \ldots 5$ considered, i.e.:

$$\Delta\mathrm{WSRT}_{\Delta E}(M) = \mathrm{WSRT}_{\Delta E}(M) - \max_{i=1\ldots5} \mathrm{WSRT}_{\Delta E}(M_i), \quad (2)$$

$$\Delta\mathrm{WSRT}_{\Delta E_{94}}(M) = \mathrm{WSRT}_{\Delta E_{94}}(M) - \max_{i=1\ldots5} \mathrm{WSRT}_{\Delta E_{94}}(M_i). \quad (3)$$

Let us define $\mathrm{MoM}_{\Delta E}(M)$ and $\mathrm{MoM}_{\Delta E_{94}}(M)$ as the mean values of the median errors obtained by the generic trans-

TABLE II. Characteristics of the corresponding color data sets used.

Data set		No. of samples	Illuminant Test	Ref.	Sample size	Medium	Experimental method
1	Lam	58	D65	A	Large	Refl.	Memory
2	Helson	59	D65	A	Small	Refl.	Memory
3	CSAJ	87	D65	A	Small	Refl.	Haploscopic
4	Lutchi	43	D65	A	Small	Refl.	Magnitude
5	Lutchi D50	44	D65	D50	Small	Refl.	Magnitude
6	Lutchi WF	41	D65	WF	Small	Refl.	Magnitude
7	Kuo&Luo	40	D65	A	Large	Refl.	Magnitude
8	Kuo&Luo TL84	41	D65	TL84	Small	Refl.	Magnitude
9	Braun&Fairchild 1	17	D65	D93	Small	Monitor&Refl.	Matching
10	Braun&Fairchild 2	16	D65	D93	Small	Monitor&Refl.	Matching
11	Braun&Fairchild 3	17	D65	D30	Small	Monitor&Refl.	Matching
12	Braun&Fairchild 4	16	D65	D30	Small	Monitor&Refl.	Matching
13	Breneman 1	12	D65	A	Small	Trans.	Magnitude
14	Breneman 8	12	D65	A	Small	Trans.	Magnitude
15	Breneman 4	12	D65	A	Small	Trans.	Magnitude
16	Breneman 6	11	D55	A	Small	Trans.	Magnitude

form M on the 16 corresponding color data sets CCDS_j, $j = 1 \ldots 16$ considered, i.e.:

$$\mathrm{MoM}_{\Delta E}(M) = \frac{\sum_{j=1}^{16} \mathrm{median}\left(\Delta E\left(\mathrm{CCDS_j}\right)\right)}{16}, \quad (4)$$

$$\mathrm{MoM}_{\Delta E_{94}}(M) = \frac{\sum_{j=1}^{16} \mathrm{median}\left(\Delta E_{94}\left(\mathrm{CCDS}_j\right)\right)}{16}. \quad (5)$$

The objective function f_{BS} we optimize is given in Eq. (6):

$$f_{\mathrm{BS}}(M) = (\Delta\mathrm{WSRT}_{\Delta E}(M) + \Delta\mathrm{WSRT}_{\Delta E_{94}}(M)) - (\mathrm{MoM}_{\Delta E}(M) + \mathrm{MoM}_{\Delta E_{94}}(M)). \quad (6)$$

The objective function f_{BS} is composed of two terms. The larger is the former, the better is the estimation of the corresponding colors given by the transformation M, according to the Wilcoxon signed-rank test. The smaller is the latter, the lower are the median errors of the transformation M on the corresponding color data sets. The new chromatic adaptation transform is found using Particle Swarm Optimization (PSO)[19] over the set $M \in \Re^{3\times3}$ of feasible solutions. PSO is a population based stochastic optimization technique which shares many similarities with evolutionary computation techniques.

A population of individuals is initialized as random guesses to the problem solutions; and a communication structure is also defined, assigning neighbors for each individual to interact with. These individuals are candidate solutions. An iterative process to improve these candidate solutions is set in motion. The particles iteratively evaluate the fitness of the candidate solutions and remember the location where they had their best success. The individual's best solution is called the particle best or the local best. Each particle makes this information available to its neighbors. They are also able to see where their neighbors have had success. Movements through the search space are guided by these successes.

The swarm is typically modeled by particles in multidimensional space that have a position and a velocity. These particles fly through hyperspace and have two essential reasoning capabilities: their memory of their own best position and their knowledge of the global or their neighborhood's best position. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions.

The new CAT M_{BS} is then defined as

$$M_{\mathrm{BS}} = \max_{M\in\Re^{3\times3}} (f_{\mathrm{BS}}(M)), \quad (7)$$

with the constraint of being equal-energy balanced.

The M_{BS} CAT that satisfies Eq. (7) is given in Eq. (8), and its normalized spectral responses are plotted in Fig. 2.

$$M_{\mathrm{BS}} = \begin{bmatrix} 0.8752 & 0.2787 & -0.1539 \\ -0.8904 & 1.8709 & 0.0195 \\ -0.0061 & 0.0162 & 0.9899 \end{bmatrix} \quad (8)$$

Following the same procedure, also a new CAT without negative lobes is found. To this end, a positivity constraint on the spectral responses corresponding to the found transform is defined as follows:

$$f_{\mathrm{PC}}(M) = \alpha \sum_{\lambda} \sum_{\text{channels}} u_{-}(\mathrm{SR}(M)),$$

where $\mathrm{SR}(M)$ are the spectral responses of the transformation M,α is a multiplicative term that reflects the importance to be given to the positivity constraint and $u_{-}(\cdot)$ is defined as

$$u_{-}(x) = \begin{cases} x & \text{if } x < 0 \\ 0 & \text{otherwise.} \end{cases}$$

The new CAT $M_{\mathrm{BS-PC}}$ is then defined as

$$M_{\mathrm{BS-PC}} = \max_{M\in\Re^{3\times3}} (f_{\mathrm{BS}}(M) + f_{\mathrm{PC}}(M)), \quad (9)$$

with the constraint of being equal-energy balanced.

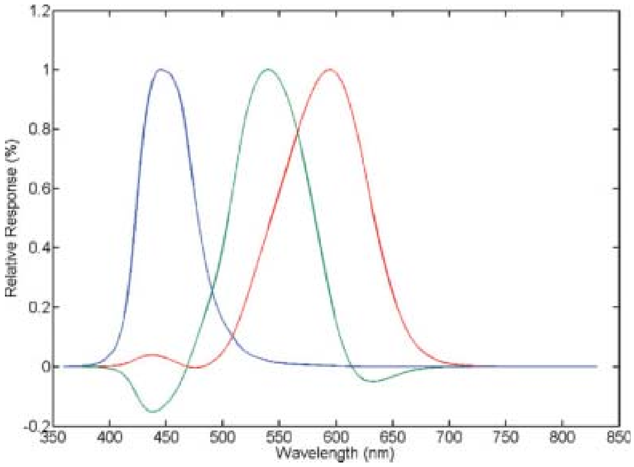

FIG. 2. The normalized spectral responses for the BS CAT. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

The M_{BS-PC} CAT that satisfies Eq. (9), fulfilling the positivity constraint, is given in Eq. (10), and its normalized spectral responses are plotted in Fig. 3.

$$M_{BS-PC} = \begin{bmatrix} 0.6489 & 0.3915 & -0.0404 \\ -0.3775 & 1.3055 & 0.0720 \\ -0.0271 & 0.0888 & 0.9383 \end{bmatrix} \quad (10)$$

PERFORMANCE EVALUATION

Predicted tristimulus values were calculated for the reference illuminants of all corresponding color data sets listed in Table II, using Eq. (1) and substituting M_{CAT} according to the specific chromatic adaptation transform tested. The actual and predicted CIE *XYZ* tristimulus values were then converted into the CIE $L^*a^*b^*$ color space. Two different perceptual error metrics, ΔE^*_{ab} and ΔE^*_{94}, were applied. Wilcoxon signed-rank tests[18] were used to compare if the variations in errors are statistically significant, as suggested by Süsstrunk and Finlayson.[20] This test is well suited to evaluate CAT performance because it does not make any assumption about the underlying error distributions, and it is easy to find out, using for example the Lilliefors test,[21] that the assumption about the normality of the error distributions does not always hold.

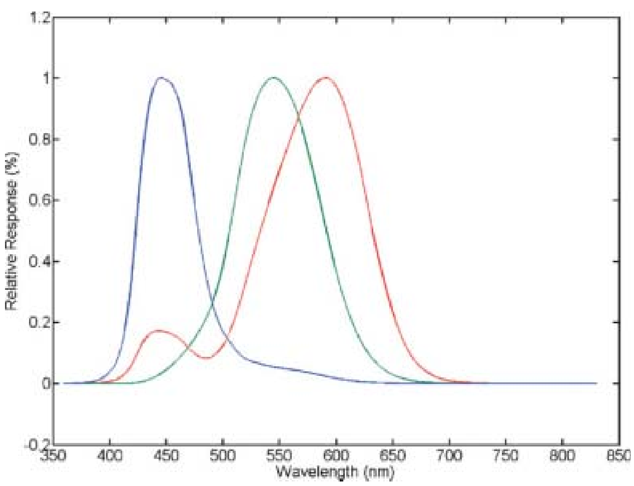

FIG. 3. The normalized spectral responses for the BS-PC CAT. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

TABLE III. A comparison among state-of-the-art transforms and the first CAT found.

Error metric	von Kries	BFD	Sharp	CMCCAT	CAT02	BS
ΔE^*_{ab}	6	11	11	11	11	16
ΔE^*_{94}	6	10	11	10	11	14

The number of times a transform performed best or was statistically the same (at the 95 percent confidence, according to the Wilcoxon signed-rank test) as the best transform.

Table III lists the number of times a transform performed best or was statistically the same as the best transform at the 95 percent confidence level, according to the Wilcoxon signed-rank test. The maximum score for each error metric is 16, as 16 corresponding color data sets were tested. As can be seen, the proposed transform, indicated as BS, outperformed existing ones.

Table IV reports the same significance test results for the positivity constrained transform proposed. As can be seen, the proposed positivity constrained transform, indicated as BS-PC, performed equally well as the best state-of-the-art transforms with negative lobes.

We report in Tables AI and AII of Appendix, a more detailed analysis of the error distribution of the investigated CATs on the corresponding color data sets considered, compared with the BS transform. In Tables AIII and AIV of Appendix, the same detailed error analysis for the BS-PC transform analysis are reported.

CONCLUSIONS

A pair of corresponding colors consists of a color observed under one set of viewing conditions that has the same appearance when observed under another set of conditions. In this article, we have proposed:

- A new von Kries based chromatic adaptation transform that outperforms existent CATs;
- A new von Kries based CAT without negative lobes that is statistically equivalent to the best available CATs;
- A new optimization procedure that simultaneously uses all the corresponding color data sets available and the

TABLE IV. A comparison among state-of-the-art transforms and the second CAT found.

Error metric	von Kries	BFD	Sharp	CMCCAT	CAT02	BS-PC
ΔE^*_{ab}	6	13	**14**	**14**	13	**14**
ΔE^*_{94}	6	11	**12**	9	**12**	**12**

The number of times a transform performed best or was statistically the same (at the 95 percent confidence, according to the Wilcoxon signed-rank test) as the best transform.

predictions of the corresponding colors done using already defined CATs.

These transforms and the optimization procedure proposed should be further evaluated both theoretically and experimentally. Our contribution should be therefore considered just in response to the needs of practical solutions that should fit the experimental data as well as possible.[7] In the proposed strategy, we used and equally treat all the different corresponding color data sets publicly available and used in other experimental comparisons of CATs.[9,7] In different application domains, it could be possible to give more importance to some data sets and less to others in the framework of the optimization procedure or apply the procedure to own datasets.

Further research will include the study of nonlinear chromatic adaptation transforms.

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APPENDIX

TABLE AI. Median and mean ΔE^*_{ab} color difference of actual and predicted colors, bold *P*-values indicate that there is 95 percent confidence (according to the Wilcoxon signed-rank test) that the transform performs as well as the best transform for a given data set.

		von Kries	BFD	Sharp	CMCCAT	CAT02	BS
Lam data set	Median ΔE^*_{ab}	5.92	**4.03**	**4.19**	**4.48**	**3.91**	**4.04**
	Mean ΔE^*_{ab}	6.50	**4.43**	**4.45**	**4.51**	**4.40**	**4.40**
	P-value	0.0000	**0.7775**	**0.6285**	**0.2198**	**n/a**	**0.8983**
Helson	Median ΔE^*_{ab}	5.71	**4.73**	**4.91**	**5.20**	**5.06**	**5.16**
	Mean ΔE^*_{ab}	6.89	**5.55**	**5.33**	**5.32**	**5.22**	**5.30**
	P-value	0.0008	**n/a**	**0.3854**	**0.2771**	**0.5560**	**0.1695**
CSAJ	Median ΔE^*_{ab}	6.38	5.16	4.73	5.16	4.79	**4.57**
	Mean ΔE^*_{ab}	6.63	5.36	5.12	5.18	5.02	**4.85**
	P-value	0.0000	0.0005	0.0042	0.0002	0.0044	**n/a**
Lutchi	Median ΔE^*_{ab}	**5.35**	**5.72**	**6.51**	**5.86**	**5.98**	**6.69**
	Mean ΔE^*_{ab}	**7.05**	**6.90**	**6.77**	**5.98**	**6.07**	**6.63**
	P-value	**n/a**	**0.4255**	**0.3980**	**0.0864**	**0.1251**	**0.3047**
Lutchi D50	Median ΔE^*_{ab}	**5.33**	5.82	**5.69**	**5.38**	**5.38**	**5.83**
	Mean ΔE^*_{ab}	**5.82**	6.32	**6.28**	**6.01**	**6.05**	**6.29**
	P-value	**n/a**	0.0051	**0.0780**	**0.4765**	**0.4008**	**0.0513**
Lutchi WF	Median ΔE^*_{ab}	9.26	7.22	7.23	6.83	6.53	**5.98**
	Mean ΔE^*_{ab}	10.55	8.87	7.80	7.54	7.27	**6.88**
	P-value	0.0000	0.0000	0.0000	0.0000	0.0004	**n/a**
Kuo & Luo	Median ΔE^*_{ab}	8.53	**6.10**	**6.78**	6.26	6.21	**5.40**
	Mean ΔE^*_{ab}	9.10	**6.37**	**6.93**	7.29	7.02	**6.65**
	P-value	0.0001	**0.6671**	**0.4930**	0.0009	0.0193	**n/a**
Kuo & Luo TL84	Median ΔE^*_{ab}	5.01	4.33	3.92	3.74	**3.71**	**3.46**
	Mean ΔE^*_{ab}	5.76	4.61	4.30	4.40	**4.27**	**4.12**
	P-value	0.0000	0.0001	0.0481	0.0090	**0.1345**	**n/a**
Braun&Fairchild 1	Median ΔE^*_{ab}	**2.81**	**3.34**	**4.00**	**3.47**	**3.61**	**4.00**
	Mean ΔE^*_{ab}	**3.60**	**3.59**	**3.76**	**3.67**	**3.82**	**3.86**
	P-value	**n/a**	**0.5228**	**0.4631**	**0.4074**	**0.2461**	**0.3088**
Braun&Fairchild 2	Median ΔE^*_{ab}	**5.54**	**5.06**	**5.02**	**5.17**	5.50	**5.22**
	Mean ΔE^*_{ab}	**6.30**	**5.96**	**5.90**	**6.06**	6.14	**6.01**
	P-value	**0.6417**	**0.4691**	**n/a**	**0.1337**	0.0131	**0.0557**
Braun&Fairchild 3	Median ΔE^*_{ab}	7.99	6.97	6.93	7.16	6.99	**6.66**
	Mean ΔE^*_{ab}	9.24	7.07	7.06	7.50	7.24	**6.52**
	P-value	0.0005	0.0148	0.0395	0.0036	0.0031	**n/a**
Braun&Fairchild 4	Median ΔE^*_{ab}	6.69	**5.99**	**6.09**	**6.01**	**5.81**	**5.67**
	Mean ΔE^*_{ab}	6.72	**5.73**	**5.92**	**6.05**	**5.92**	**5.76**
	P-value	0.0229	**0.3794**	**0.1337**	**0.1208**	**0.2775**	**n/a**
Breneman 1	Median ΔE^*_{ab}	**9.70**	**8.70**	**10.23**	**10.04**	**10.06**	**8.87**
	Mean ΔE^*_{ab}	**10.72**	**9.10**	**10.53**	**10.14**	**9.68**	**9.27**
	P-value	**0.0923**	**n/a**	**0.1514**	**0.7334**	**0.8501**	**0.9697**
Breneman 8	Median ΔE^*_{ab}	12.83	**14.10**	**12.06**	**11.01**	**11.02**	**10.88**
	Mean ΔE^*_{ab}	16.32	**14.04**	**12.05**	**11.79**	**11.25**	**11.64**
	P-value	0.0161	**0.1763**	**0.2661**	**0.1294**	**0.9697**	**n/a**
Breneman 4	Median ΔE^*_{ab}	14.62	**14.93**	**11.15**	**10.20**	**10.27**	**10.68**
	Mean ΔE^*_{ab}	17.35	**14.67**	**12.27**	**12.01**	**11.61**	**12.11**
	P-value	0.0093	**0.2661**	**0.6772**	**n/a**	**0.2661**	**0.4238**
Breneman 6	Median ΔE^*_{ab}	**6.94**	**7.13**	6.90	**6.75**	**6.76**	**6.88**
	Mean ΔE^*_{ab}	**7.38**	**7.73**	7.92	**6.81**	**6.83**	**7.45**
	P-value	**0.8311**	**0.1230**	0.0068	**n/a**	**0.9658**	**0.1016**

TABLE AII. Median and mean ΔE^*_{94} color difference of actual and predicted colors, bold *P*-values indicate that there is 95 percent confidence (according to the Wilcoxon signed-rank test) that the transform performs as well as the best transform for a given data set.

		von Kries	BFD	Sharp	CMCCAT	CAT02	BS
Lam Data Set	Median ΔE^*_{94}	3.98	**2.76**	**2.70**	**2.84**	**2.62**	**2.56**
	Mean ΔE^*_{94}	4.31	**3.00**	**2.93**	**3.03**	**2.97**	**2.93**
	P-value	0.0000	**0.9660**	**0.8617**	**0.1532**	**0.3630**	**n/a**
Helson	Median ΔE^*_{94}	4.03	**2.77**	**2.89**	**3.04**	**3.02**	**3.06**
	Mean ΔE^*_{94}	4.52	**3.49**	**3.42**	**3.55**	**3.45**	**3.45**
	P-value	0.0001	**n/a**	**0.8444**	**0.0463**	**0.5409**	**0.5460**
CSAJ	Median ΔE^*_{94}	4.56	3.44	3.45	3.58	3.28	**3.18**
	Mean ΔE^*_{94}	4.71	3.84	3.72	3.76	3.66	**3.58**
	P-value	0.0000	0.0004	0.0017	0.0011	0.0117	**n/a**
Lutchi	Median ΔE^*_{94}	**3.41**	3.58	4.30	**2.89**	3.05	3.39
	Mean ΔE^*_{94}	**3.47**	3.71	4.03	**3.10**	3.35	3.83
	P-value	**0.1613**	0.0003	0.0000	**n/a**	0.0000	0.0000
Lutchi D50	Median ΔE^*_{94}	**2.23**	3.20	3.48	3.29	3.41	3.63
	Mean ΔE^*_{94}	**3.11**	3.52	3.59	3.40	3.45	3.61
	P-value	**n/a**	0.0003	0.0012	0.0084	0.0042	0.0006
Lutchi WF	Median ΔE^*_{94}	4.69	4.09	3.93	3.62	3.48	**3.37**
	Mean ΔE^*_{94}	5.61	4.44	3.97	3.96	3.85	**3.60**
	P-value	0.0000	0.0000	0.0000	0.0000	0.0004	**n/a**
Kuo & Luo	Median ΔE^*_{94}	4.55	**4.00**	**4.18**	**3.79**	**3.81**	**3.77**
	Mean ΔE^*_{94}	5.26	**3.87**	**3.98**	**3.95**	**3.99**	**3.98**
	P-value	0.0003	**0.4597**	**0.8297**	**0.9464**	**0.7368**	**n/a**
Kuo & Luo TL84	Median ΔE^*_{94}	3.03	2.73	2.66	**2.53**	**2.36**	**2.18**
	Mean ΔE^*_{94}	3.34	2.79	2.71	**2.67**	**2.64**	**2.59**
	P-value	0.0005	0.0004	0.0281	**0.2207**	**0.5129**	**n/a**
Braun&Fairchild 1	Median ΔE^*_{94}	**2.24**	**2.52**	**2.76**	**2.53**	**2.64**	**2.73**
	Mean ΔE^*_{94}	**2.72**	**2.68**	**2.79**	**2.71**	**2.71**	**2.78**
	P-value	**n/a**	**0.5862**	**0.3088**	**0.4925**	**0.5228**	**0.4348**
Braun&Fairchild 2	Median ΔE^*_{94}	**3.89**	**3.38**	**3.38**	3.46	**3.41**	**3.41**
	Mean ΔE^*_{94}	**4.73**	**4.53**	**4.50**	4.58	**4.63**	**4.56**
	P-value	**0.8767**	**n/a**	**0.6791**	0.0052	**0.0557**	**0.5014**
Braun&Fairchild 3	Median ΔE^*_{94}	5.95	4.68	**4.38**	4.88	4.79	**4.05**
	Mean ΔE^*_{94}	5.99	4.53	**4.27**	4.75	4.59	**4.21**
	P-value	0.0010	0.0129	**0.2659**	0.0065	0.0065	**n/a**
Braun&Fairchild 4	Median ΔE^*_{94}	4.42	**3.95**	**4.06**	**4.04**	**3.92**	**4.13**
	Mean ΔE^*_{94}	4.76	**4.03**	**3.97**	**4.18**	**4.11**	**4.03**
	P-value	0.0113	**0.2146**	**0.5695**	**0.3011**	**n/a**	**0.5014**
Breneman 1	Median ΔE^*_{94}	**4.10**	**4.27**	**5.09**	**4.12**	**4.31**	**4.60**
	Mean ΔE^*_{94}	**5.46**	**5.02**	**5.57**	**4.97**	**5.13**	**5.30**
	P-value	**n/a**	**0.9697**	**0.7910**	**0.9097**	**0.8501**	**0.8501**
Breneman 8	Median ΔE^*_{94}	6.35	**6.78**	**7.18**	**6.00**	**5.86**	**6.00**
	Mean ΔE^*_{94}	8.48	**7.17**	**6.83**	**6.46**	**6.47**	**6.58**
	P-value	0.0210	**0.1514**	**0.3394**	**0.7334**	**n/a**	**0.4238**
Breneman 4	Median ΔE^*_{94}	6.23	**6.37**	**6.38**	**5.28**	**5.17**	**5.30**
	Mean ΔE^*_{94}	9.45	**7.86**	**7.24**	**7.05**	**7.01**	**7.08**
	P-value	0.0093	**0.0522**	**0.5186**	**0.9697**	**n/a**	**0.6221**
Breneman 6	Median ΔE^*_{94}	**3.31**	**3.69**	**4.30**	**3.69**	**4.12**	**3.90**
	Mean ΔE^*_{94}	**3.68**	**4.17**	**4.66**	**3.86**	**4.08**	**4.39**
	P-value	**n/a**	**0.5771**	**0.4131**	**0.7646**	**0.6377**	**0.6377**

TABLE AIII. Median and mean ΔE^*_{ab} color difference of actual and predicted colors, bold *P*-values indicate that there is 95 percent confidence (according to the Wilcoxon signed-rank test) that the transform performs as well as the best transform for a given data set.

		von Kries	BFD	Sharp	CMCCAT	CAT02	BS-PC
Lam Data Set	Median ΔE^*_{ab}	5.92	**4.03**	**4.19**	**4.48**	**3.91**	4.71
	Mean ΔE^*_{ab}	6.50	**4.43**	**4.45**	**4.51**	**4.40**	5.14
	P-value	0.0000	**0.7775**	**0.6285**	**0.2198**	**n/a**	0.0004
Helson	Median ΔE^*_{ab}	5.71	**4.73**	**4.91**	**5.20**	**5.06**	5.89
	Mean ΔE^*_{ab}	6.89	**5.55**	**5.33**	**5.32**	**5.22**	6.19
	P-value	0.0008	**n/a**	**0.3854**	**0.2771**	**0.556**	0.0006
CSAJ	Median ΔE^*_{ab}	6.38	**5.16**	**4.73**	**5.16**	**4.79**	**5.04**
	Mean ΔE^*_{ab}	6.63	**5.36**	**5.12**	**5.18**	**5.02**	**5.34**
	P-value	0.0000	**0.2428**	**n/a**	**0.3949**	**0.0741**	**0.0790**

TABLE AIII. (Continued)

		von Kries	BFD	Sharp	CMCCAT	CAT02	BS-PC
Lutchi	Median ΔE^*_{ab}	**5.35**	**5.72**	**6.51**	**5.86**	**5.98**	**5.65**
	Mean ΔE^*_{ab}	**7.05**	**6.90**	**6.77**	**5.98**	**6.07**	**6.81**
	P-value	**n/a**	**0.4255**	**0.398**	**0.0864**	**0.1251**	**0.4837**
Lutchi D50	Median ΔE^*_{ab}	**5.33**	5.82	**5.69**	**5.38**	**5.38**	**5.76**
	Mean ΔE^*_{ab}	**5.82**	6.32	**6.28**	**6.01**	**6.05**	**6.34**
	P-value	**n/a**	0.0051	**0.0780**	**0.4765**	**0.4008**	**0.0513**
Lutchi WF	Median ΔE^*_{ab}	9.26	7.22	7.23	6.83	6.53	**6.05**
	Mean ΔE^*_{ab}	10.55	8.87	7.80	7.54	7.27	**6.50**
	P-value	0.0000	0.0000	0.0000	0.0000	0.0000	**n/a**
Kuo & Luo	Median ΔE^*_{ab}	8.53	**6.10**	**6.78**	**6.26**	**6.21**	**6.83**
	Mean ΔE^*_{ab}	9.10	**6.37**	**6.93**	**7.29**	**7.02**	**7.80**
	P-value	0.0000	**n/a**	**0.2532**	**0.1068**	**0.3750**	**0.0783**
Kuo & Luo TL84	Median ΔE^*_{ab}	5.01	4.33	**3.92**	**3.74**	**3.71**	**3.47**
	Mean ΔE^*_{ab}	5.76	4.61	**4.30**	**4.40**	**4.27**	**4.27**
	P-value	0.0000	0.0084	**0.7313**	**0.1124**	**0.9948**	**n/a**
Braun&Fairchild 1	Median ΔE^*_{ab}	**2.81**	**3.34**	**4.00**	**3.47**	**3.61**	**3.82**
	Mean ΔE^*_{ab}	**3.60**	**3.59**	**3.76**	**3.67**	**3.82**	**3.81**
	P-value	**n/a**	**0.5228**	**0.4631**	**0.4074**	**0.2461**	**0.3318**
Braun&Fairchild 2	Median ΔE^*_{ab}	**5.54**	**5.06**	**5.02**	**5.17**	5.50	**4.94**
	Mean ΔE^*_{ab}	**6.30**	**5.96**	**5.90**	**6.06**	6.14	**5.97**
	P-value	**0.6791**	**0.8767**	**0.9176**	**0.0627**	0.0494	**n/a**
Braun&Fairchild 3	Median ΔE^*_{ab}	7.99	**6.97**	**6.93**	7.16	**6.99**	**6.75**
	Mean ΔE^*_{ab}	9.24	**7.07**	**7.06**	7.50	**7.24**	**7.19**
	P-value	0.0010	**0.6874**	**0.8313**	0.0495	**0.4925**	**n/a**
Braun&Fairchild 4	Median ΔE^*_{ab}	6.69	**5.99**	**6.09**	**6.01**	**5.81**	**5.97**
	Mean ΔE^*_{ab}	6.72	**5.73**	**5.92**	**6.05**	**5.92**	**6.24**
	P-value	0.0200	**0.3011**	**0.7564**	**0.352**	**n/a**	**0.8361**
Breneman 1	Median ΔE^*_{ab}	**9.70**	**8.70**	**10.23**	**10.04**	**10.06**	**10.50**
	Mean ΔE^*_{ab}	**10.72**	**9.10**	**10.53**	**10.14**	**9.68**	**11.1**
	P-value	**0.0923**	**n/a**	**0.1514**	**0.7334**	**0.8501**	**0.2334**
Breneman 8	Median ΔE^*_{ab}	12.83	**14.10**	**12.06**	**11.01**	11.02	**11.17**
	Mean ΔE^*_{ab}	16.32	**14.04**	**12.05**	**11.79**	11.25	**12.29**
	P-value	0.0342	**0.4238**	**0.9697**	**n/a**	0.0425	**0.2036**
Breneman 4	Median ΔE^*_{ab}	14.62	**14.93**	**11.15**	**10.2**	**10.27**	**10.39**
	Mean ΔE^*_{ab}	17.35	**14.67**	**12.27**	**12.01**	**11.61**	**12.23**
	P-value	0.0093	**0.2661**	**0.6772**	**n/a**	**0.2661**	**0.3804**
Breneman 6	Median ΔE^*_{ab}	**6.94**	**7.13**	6.90	**6.75**	**6.76**	**6.94**
	Mean ΔE^*_{ab}	**7.38**	**7.73**	7.92	**6.81**	**6.83**	**7.41**
	P-value	**0.8311**	**0.123**	0.0068	**n/a**	**0.9658**	**0.2061**

TABLE AIV. Median and mean ΔE^*_{94} color difference of actual and predicted colors, bold P-values indicate that there is 95 percent confidence (according to the Wilcoxon signed-rank test) that the transform performs as well as the best transform for a given data set.

		von Kries	BFD	Sharp	CMCCAT	CAT02	BS-PC
Lam Data Set	Median ΔE^*_{94}	3.98	**2.76**	**2.70**	**2.84**	**2.62**	3.32
	Mean ΔE^*_{94}	4.31	**3**	**2.93**	**3.03**	**2.97**	3.41
	P-value	0.0000	**0.8861**	**0.8556**	**0.1974**	**n/a**	0.0017
Helson	Median ΔE^*_{94}	4.03	**2.77**	**2.89**	3.04	**3.02**	3.71
	Mean ΔE^*_{94}	4.52	**3.49**	**3.42**	3.55	**3.45**	4.19
	P-value	0.0001	**n/a**	**0.8444**	0.0463	**0.5409**	0.0000
CSAJ	Median ΔE^*_{94}	4.56	3.44	**3.45**	3.58	**3.28**	3.51
	Mean ΔE^*_{94}	4.71	3.84	**3.72**	3.76	**3.66**	3.85
	P-value	0.0000	0.0115	**0.2549**	0.0003	**n/a**	0.0010
Lutchi	Median ΔE^*_{94}	**3.41**	3.58	4.30	**2.89**	3.05	**3.00**
	Mean ΔE^*_{94}	**3.47**	3.71	4.03	**3.10**	3.35	**3.32**
	P-value	**0.1613**	0.0003	0.0000	**n/a**	0.0000	**0.1473**
Lutchi D50	Median ΔE^*_{94}	**2.23**	3.20	3.48	3.29	3.41	3.51
	Mean ΔE^*_{94}	**3.11**	3.52	3.59	3.40	3.45	3.59
	P-value	**n/a**	0.0003	0.0012	0.0084	0.0042	0.0016
Lutchi WF	Median ΔE^*_{94}	4.69	4.09	3.93	3.62	3.48	**3.41**
	Mean ΔE^*_{94}	5.61	4.44	3.96	3.96	3.85	**3.41**
	P-value	0.0000	0.0000	0.0000	0.0000	0.0000	**n/a**
Kuo & Luo	Median ΔE^*_{94}	4.55	**4.00**	**4.18**	**3.79**	**3.81**	**3.57**
	Mean ΔE^*_{94}	5.26	**3.87**	**3.98**	**3.95**	**3.99**	**4.02**
	P-value	0.0046	**0.7368**	**0.8402**	**0.5724**	**0.6477**	**n/a**

TABLE AIV. (Continued)

		von Kries	BFD	Sharp	CMCCAT	CAT02	BS-PC
Kuo & Luo TL84	Median ΔE^*_{94}	3.03	2.73	2.66	2.53	2.36	**2.36**
	Mean ΔE^*_{94}	3.34	2.79	2.71	2.67	2.64	**2.51**
	P-value	0.0001	0.0003	0.0290	0.0018	0.0214	**n/a**
Braun&Fairchild 1	Median ΔE^*_{94}	**2.24**	**2.52**	**2.76**	**2.53**	**2.64**	**2.76**
	Mean ΔE^*_{94}	**2.72**	**2.68**	**2.79**	**2.71**	**2.71**	**2.91**
	P-value	**n/a**	**0.5862**	**0.3088**	**0.4925**	**0.5228**	**0.1024**
Braun&Fairchild 2	Median ΔE^*_{94}	**3.89**	**3.38**	**3.38**	3.46	**3.41**	**3.45**
	Mean ΔE^*_{94}	**4.73**	**4.53**	**4.50**	4.58	**4.63**	**4.53**
	P-value	**0.8767**	**n/a**	**0.6791**	0.0052	**0.0557**	**0.9999**
Braun&Fairchild 3	Median ΔE^*_{94}	5.95	**4.68**	**4.38**	4.88	**4.79**	**4.66**
	Mean ΔE^*_{94}	5.99	**4.53**	**4.27**	4.75	**4.59**	**4.79**
	P-value	0.0007	**0.0684**	**n/a**	0.0056	**0.0840**	**0.2461**
Braun&Fairchild 4	Median ΔE^*_{94}	4.42	**3.95**	**4.06**	**4.04**	**3.92**	**4.17**
	Mean ΔE^*_{94}	4.76	**4.03**	**3.97**	**4.18**	**4.11**	**4.41**
	P-value	0.0113	**0.2146**	**0.5695**	**0.3011**	**n/a**	**0.2775**
Breneman 1	Median ΔE^*_{94}	**4.10**	**4.27**	**5.09**	**4.12**	**4.31**	**4.81**
	Mean ΔE^*_{94}	**5.46**	**5.02**	**5.57**	**4.97**	**5.13**	**5.21**
	P-value	**n/a**	**0.9697**	**0.791**	**0.9097**	**0.8501**	**0.9697**
Breneman 8	Median ΔE^*_{94}	6.35	**6.78**	**7.18**	**6.00**	**5.86**	**6.31**
	Mean ΔE^*_{94}	8.48	**7.17**	**6.83**	**6.46**	**6.47**	**6.29**
	P-value	0.0210	**0.1514**	**0.3394**	**0.7334**	**n/a**	**0.791**
Breneman 4	Median ΔE^*_{94}	6.23	**6.37**	**6.38**	**5.28**	**5.17**	**5.94**
	Mean ΔE^*_{94}	9.45	**7.86**	**7.24**	**7.05**	**7.01**	**6.83**
	P-value	0.0093	**0.0522**	**0.5186**	**0.9697**	**n/a**	**0.6221**
Breneman 6	Median ΔE^*_{94}	**3.31**	**3.69**	**4.30**	**3.69**	**4.12**	**4.01**
	Mean ΔE^*_{94}	**3.68**	**4.17**	**4.66**	**3.86**	**4.08**	**3.99**
	P-value	**n/a**	**0.5771**	**0.4131**	**0.7646**	**0.6377**	**0.5771**

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